

An Investigation of the Homopolar Generator:

Determining the Impact of System Characteristics on Efficiency

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Abstract—We examine the effect of mechanical and electrical characteristics on the output of a homopolar generator, and investigate the conditions that result in the maximum efficiency of the generator. A comprehensive model that takes into account most of the experimental variables is presented and validated by considering the limited conditions. The results of the simulations indicate that maximum efficiency can be obtained by increasing the strength of the magnetic field and by decreasing the coefficient of viscous friction and resistivity. Increasing initial angular velocity cannot increase maximum efficiency—however, it must be beyond a certain value in order to overcome the effects of dry friction.

I. INTRODUCTION

A homopolar generator, also known as unipolar or acyclic generator, was first suggested by Michael Faraday [1] back in 1831. He discovered that a cylindrical magnet suspended by a string and touching a mercury bath at the bottom could generate electricity while spinning along its axis if a second electrical contact was made at the periphery of the mid-point of the magnet. The homopolar generator distinguishes itself from other common generators in that no commutation or alternating of the magnetic poles is necessary for this machine in order to generate electricity. Due to the exceptionally high theoretical efficiency a homopolar generator might produce, there have been a number of experimental attempts to build the most ideal homopolar generator, especially since the early 20th century when the significance of DC generator was highlighted. Thomas Valone [2] is perhaps the most prominent researcher in the field of the modern homopolar generator, whose primary focus was to reduce the power loss due to the brush contact. Beaty [3] proposes a numerous different designs of homopolar generators, some of which are left untested, including the bilateral symmetry model and the self-excited cylindrical model.

Although there have been many experimental approaches to find an efficient homopolar generator, very few attempts have been conducted to develop a mechanical and electrical model of a homopolar generator. In this article, we focus on modeling the electrical behavior of a homopolar generator depending on various system characteristics. We intend to investigate the conditions where the maximum efficiency can be obtained based on simulation results. This research is significant in that it suggests a general strategy in building a homopolar generator under the physical and experimental constraints.

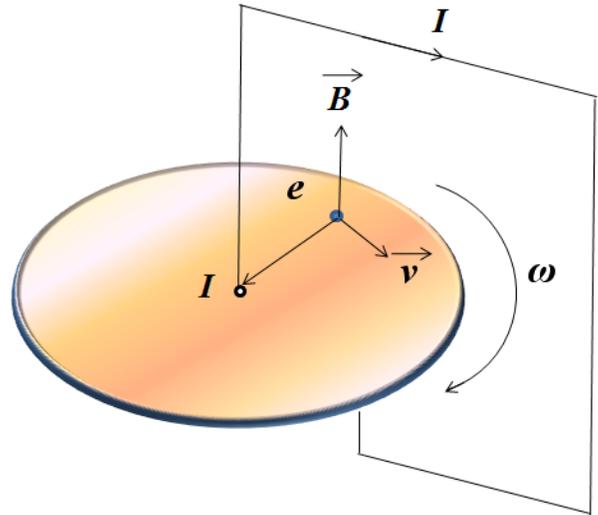


Fig. 1: Illustration of how a homopolar generator creates the motional emf in the presence of magnetic field perpendicular to a conductor.

II. THE MODEL

The emf generated in the rotating disk is fundamentally due to the Lorentz force on the electrons in the moving conductor placed in a magnetic field. The electrons move with the initial angular velocity ω of the conductor and in the presence of a magnetic field B . The electric field due to the Lorentz force acting on an electron can be expressed as:

$$\vec{E} = (\vec{\omega} \times \vec{r}) \times \vec{B} \quad (1)$$

where \vec{E} is the induced electric field. Thus in Figure 1, the electrons are moved to the rim of the disk and an emf is generated between the rim and axis of the disk. However, for the sake of simplicity, the motion of the electrons is not considered in this model, since it is known that the electrons barely move in a conductor when it creates current. (Figure 2)

Therefore,

$$V = \int_0^R \vec{E} \cdot d\vec{r} \quad (2)$$

The two governing factors that decelerate the rotation of the conductor are the physical friction and the armature reaction.

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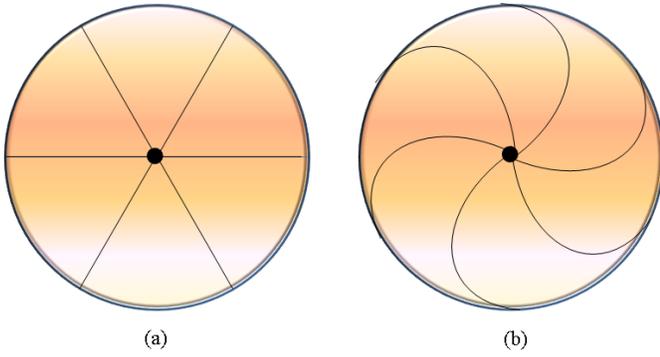


Fig. 2: Illustration of how the current is formed in a disk (a) when the radial motion of the electrons is ignored and (b) when it is taken account. Due to the Lorentz force acting on an electron, the electrons are forced to the rim of the disk. When the angular velocity is higher, this causes a spiral-shaped current on the disk. However, the movement of electrons is not considered in our model, causing it to only be accurate in situations of low angular velocities.

We model the frictional torque acting on the conductor through the following equation:

$$\vec{f} = \beta\omega + \alpha \cdot \text{sign}(\omega) \quad (3)$$

where f is the frictional torque on the rotating disk, β is the coefficient of viscous friction, and α is the coefficient of dry kinetic friction.

Another major factor that resists the rotation of the disk of the generator is the armature reaction, or commonly referred as a back torque. In the case of a homopolar generator, the back torque is simply due to the Lorentz force by the induced current:

$$d\vec{\tau} = \vec{r} \times (I d\vec{r} \times \vec{B}) \quad (4)$$

where B is the magnetic field at the certain point r on the disk, and I is the current on the disk induced by a generator. Therefore, the equation for the total back torque acting on the disk can be expressed as:

$$\tau = \int_0^R B I r \cdot dr \quad (5)$$

where R is the radius of the disk. Note that B and I is also dependent on r . Note also that the resulting back torque affects the magnitude of the induced current. Therefore, the simulation model is designed to consequently update the parameters and the variables every time it is run.

Lastly, the efficiency of the generator can be defined as the total output energy divided by the input energy. In order to measure the efficiency, we set a simulation where the disk is first actuated by a stroke of a given input energy, and allowed to decelerate slowly. The generator will produce power, which can be integrated with respect to the time to calculate the output energy. Thus, the efficiency can be expressed as:

$$\eta = \frac{\int P dt}{\frac{1}{2} I \omega_0^2} \quad (6)$$

where η is the efficiency of the homopolar generator, P is the induced power, I is the rotational inertia of the conductor, and ω_0 is the initial angular velocity. $\frac{1}{2} I \omega_0^2$ thus refers to the initial mechanical energy. In order to determine the power produced as a function of time, we can express $P(t)$ as $V(t)^2/R_\Omega$ where R_Ω is the resistance of the disk. In order to calculate resistance for a disk, we will start with a simpler example.

The resistance of a linear wire is known to be:

$$R_\Omega = \frac{\rho}{A} \int dl \quad (7)$$

where R_Ω is the resistance of a wire, ρ is the resistivity of the material of the wire, A is the cross sectional area of the wire, and L is the length of the wire.

Assuming the direction of the current is radial, the resistance of a disk-shaped conductor can be calculated by considering an infinitesimal fraction of a circle (an arc) as a thin wire, of which the cross sectional area A is dependent on L . Therefore, the resistance of a disk-shaped conductor can be expressed as:

$$R_\Omega = \rho \int_0^{2\pi} \frac{dl}{\int_0^{2\pi} dl \theta h} \quad (8)$$

where l is the radial distance from the center of the disk, h is the thickness of the disk. By solving the equation, we obtain:

$$R_\Omega = \frac{\rho}{2\pi h} (\ln(R_O) - \ln(R_I)) \quad (9)$$

where R_O is the radius of the disk, R_I is the inner radius.

A limitation of this model is that it does not accurately describe a situation when a current with a strong angular component is produced on the disk due to a high angular velocity, or with a high overall current. Thus the change in the magnetic flux along the path of the electrons creates an eddy current, which resists the rotation of the disk as a consequence.

III. RESULTS AND ANALYSIS

In all of our simulations, the disk is given an initial angular velocity and allowed to spin freely. As the disk spins, frictional forces and the armature reaction act against the angular momentum of the disk, eventually bringing it to a stop. Choosing to allow the disk to spin freely after given an initial rotational energy rather than providing it with an external power input yields several analytical advantages. As the energy leaving the system (through current generation or friction) will most likely not equal the energy entering the system, it will take some time before a steady state can be reached—allowing efficiency to be calculated. In the case of a disk with a relatively high resistance and low (near-zero) coefficients of friction, the armature reaction will be extremely small. The steady state of such a disk will take a fair amount of time to reach and could possibly be at relativistic speeds. Also, at extremely high angular velocities,

the effects of eddy currents become increasingly powerful. As the inclusion of eddy currents and relativistic effects are beyond the scope of our model, we will instead use our model to explore the efficiency of the disk at lower angular velocities while varying other system characteristics greatly. In order to calculate efficiency in our simulations, we will determine efficiency by the ratio between the output energy ($\int_0^{T_{stop}} P(t)dt$) and the input energy ($0.5I\omega^2$). Lastly, for the sake of consistency, all variables that are not varied will remain the same for all of our simulations (unless otherwise specified). The initial angular velocity will be 10 radians per second, the radius of the disk will be 1m, the thickness of the disk will be 1mm, the density and electrical resistivity of the disk will be that of pure copper, and the inner radius of the disk will be 1mm.

To test the validity of our model, we simulated a known limiting case—one of zero friction. As our model does not include electrical energy loss due to eddy currents (as we test only relatively low angular velocities), the efficiency of our system should be 100%. We varied the initial angular velocity, the strength of the magnetic field, the radius, mass, thickness, and resistivity of the disk, and found that in each zero-friction case the efficiency was, in fact, 100%. Based on the fact that the presence of dry and/or viscous friction characterizes all systems with less than 100% efficiency in our model, we will show the effect of all tested system parameters in relation to varying dry and viscous friction values. It should be noted that, even though the disk will never actually stop without friction, we will assume that the disk has “stopped” when its angular velocity is less than 0.0001 radians per second.

Even though the overall efficiency did not change when varying different system characteristics in the no-friction case, we did find important variations in the amount of time the disk takes to stop spinning. The more time that it takes the disk to convert its rotational energy into electrical energy, the greater the energy loss to frictional forces acting on the system. Also, the more time that the disk is at an elevated angular velocity, the greater the energy loss to eddy currents (although this is not shown in our simulations.)

If we examine the relationship between the resistivity ρ_e of the disk and the time it takes for the disk to stop spinning, as seen in figure 3, we will observe the following relationship:

$$t_{stop} \propto \rho_e^2 \quad (10)$$

This is because the current produced for a particular emf is inversely proportional to the resistance. As the disk is only slowing down when power is produced, and power is proportional to the square of current, an increase in resistivity will lead to a squared decrease in the amount of power generated. As the rate of power generation in the zero friction case is proportional to the acceleration of the disk, the amount of time the disk spins will be increased as the power generation decreases. Therefore, we find that the relationship expressed in equation 10 deduced from figure 3 to be correct.

If we examine the relationship between the initial angular velocity of the disk and its voltage over time curve, as seen in figure 4, we find that the shape of the curve is unaffected

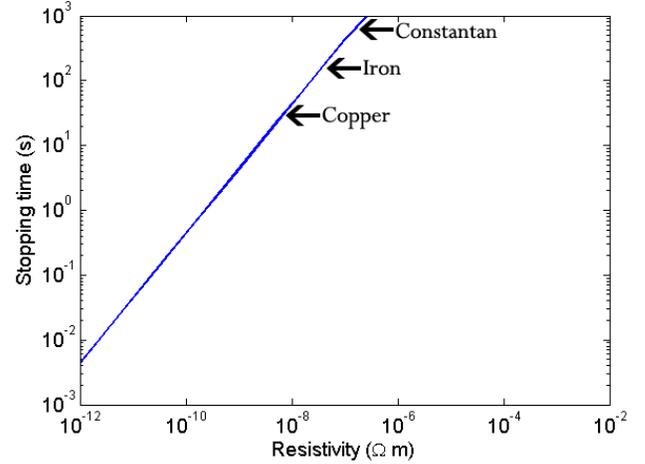


Fig. 3: Time elapsed before the disk stops spinning as a function of resistivity of the copper disk. It should be noted that for every one power of ten increase in resistivity, there is an increase by two powers of ten in the amount of time it takes for the disk to stop. The resistivities of several common metals have been noted, including copper—the material used in our simulations.

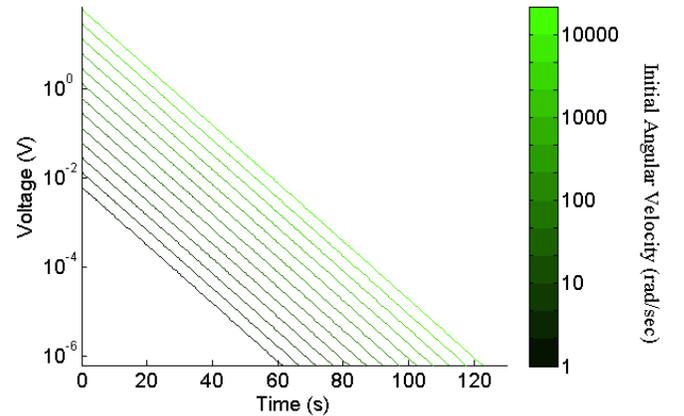


Fig. 4: Voltage (emf) produced over time for a given angular velocity. The shade of the line corresponds to the colorbar on the right indicating angular velocity. Note that the behavior for all initial conditions is the same—meaning that the same ratio of angular velocity to initial angular velocity (which is directly proportional to voltage) is present at each time, regardless of initial angular velocity. Also note that in the range of initial angular velocities which we test, significant power generation stops after approximately 40 seconds.

by the initial angular velocity, and that there is a direct relationship between initial angular velocity and voltage (as would be expected from equations 1 and 2). We should also note that the time at which significant power generation stops is approximately 40 seconds for angular velocities of less than 10,000 radians per second, and that the stoppage time only increases by a factor of 2 across this range. From this, we can infer that the amount of time during which friction can act on

the system is almost the same regardless of increased angular velocity.

A. Varying Initial Angular Velocity

If we examine the relationship between initial angular velocity of the disk and efficiency for a given value of the coefficient of dry friction α (when the coefficient of viscous friction β is zero), as seen in figure 5, we can see that the general relationship is linear in the log scale until the efficiency approaches 100%—the horizontal asymptote at infinity. This asymptote is the same for all values of dry friction. This is due to the fact that, at higher initial angular velocities, the effect of dry friction becomes increasingly less significant. In the linear region of the curve, there is an even spacing between the linear regions of the curves at different coefficients of dry friction, indicating the following relationship:

$$\eta(\omega_0) \propto \frac{1}{\alpha} \quad (11)$$

If one were to take the limit of equation 11 at infinity, however, η would approach infinity. As seen in figure 5, this relationship begins to break down at different values of angular velocity depending on the coefficient of dry friction. The relationship between the maximum angular velocity for the linear region of the curve and the coefficient of dry friction is linear. As such, the bounds of equation 11 can be expressed in the following manner:

$$(0, w_{max}] \dots \text{where } w_{max} \propto \alpha \quad (12)$$

As the maximum angular velocity for which equation 11 applies decreases as α decreases, we can deduce that there will be no linear region as α approaches zero. When dry friction is not present (and viscous friction is also zero), efficiency will be 100% regardless of initial angular velocity. However, 100% efficiency can also be approached as the angular velocity exceeds the bounds of equation 11 given in equation 12. At higher angular velocities, as stated previously, the effect of dry friction becomes relatively insignificant.

If we examine the relationship between initial angular velocity of the disk and efficiency for a given value of the coefficient of viscous friction β (when the coefficient of dry friction α is zero), as seen in figure 6, we can see that the general relationship between efficiency and angular velocity is constant for a given coefficient of viscous friction. Rather than change the overall shape of the curve as the coefficient of dry friction does, the coefficient of viscous friction determines the asymptote of the efficiency. At higher coefficients of viscous friction, the relationship between the maximum efficiency and the coefficient of viscous friction is as follows:

$$\eta_{max} \propto \frac{1}{\beta} \quad (13)$$

However, if we take the limit of this equation as with equation 11, we find that it predicts a maximum efficiency of infinity. As with equation 11, this relationship begins to

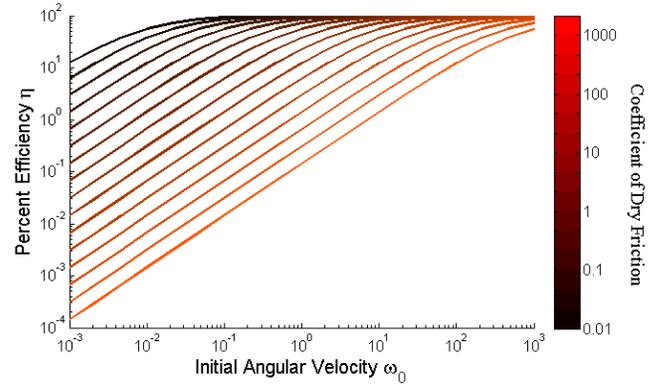


Fig. 5: Percent efficiency versus initial angular velocity for different coefficients of dry friction (zero viscous friction). Note that the coefficient of dry friction does not affect the maximum efficiency. Also note that there is a linear region (in the log scale) of the efficiency versus angular velocity curve which varies in range for a given value of dry friction.

break down as the predicted η_{max} approaches 100%. In figure 6 we can see that this relationship seems to no longer hold true at approximately 10¹% efficiency. As such, this formula holds true for coefficients of angular velocity above approximately 10. As β approaches zero, η_{max} approaches 100% as would be expected.

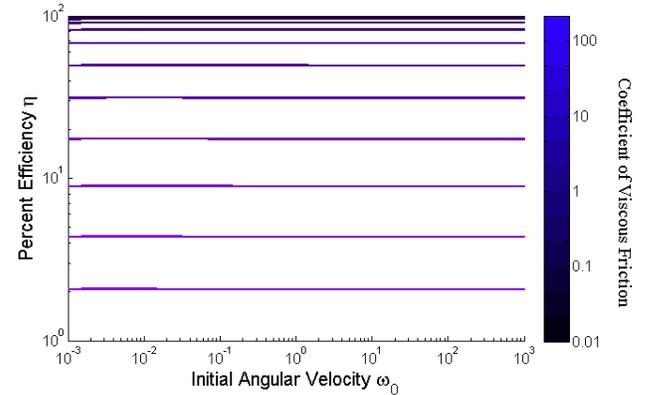


Fig. 6: Percent efficiency versus initial angular velocity for different coefficients of viscous friction (zero dry friction). Note that the coefficient of viscous friction determines the maximum efficiency.

B. Varying Magnetic Field Strength

For these simulations, we increased the amount of current running through our electromagnet. As there is a direct relationship between the current running through the coiled wires and the magnetic field produced, we can correlate an increase in this current to a relative magnetic field strength measurement. Since the magnetic field strength determines the amount of current produced, we would expect that a lower magnetic field strength for a given angular velocity would

create a situation where friction has more time to act on the system, thus decreasing efficiency. In figures 7 and 8, we can see the relationship between relative magnetic field strength and efficiency with different values of α and β respectively.

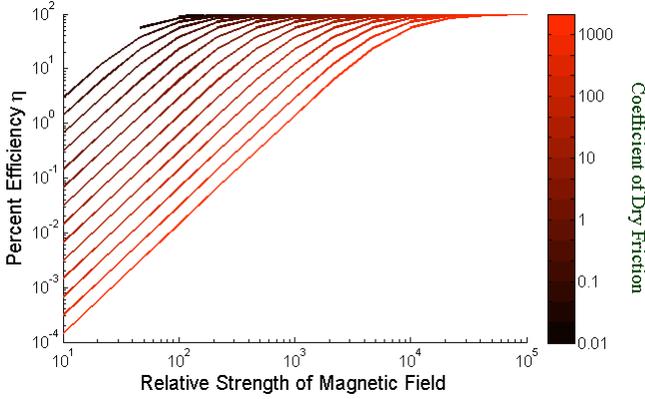


Fig. 7: Percent efficiency versus relative magnetic field strength for different coefficients of dry friction (zero viscous friction). Note that the frictional component does not determine the maximum theoretical efficiency, but that it does determine what field strength is required to reach this maximum. There is also a linear region of the curve at lower efficiencies.

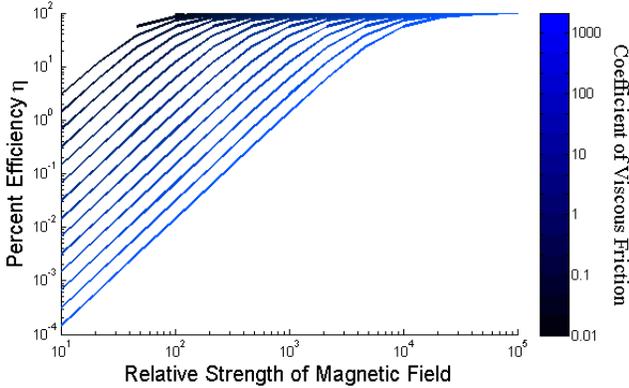


Fig. 8: Percent efficiency versus relative magnetic field strength for different coefficients of viscous friction (zero dry friction). Note that the frictional component does not determine the maximum theoretical efficiency, but that it does determine what field strength is required to reach this maximum. There is also a linear region of the curve at lower efficiencies.

For both dry and viscous friction, there is a linear region (in the log scale) of the curve where the following relationship is true:

$$\eta(B) \propto \frac{1}{\alpha} \quad (14)$$

$$\eta(B) \propto \frac{1}{\beta} \quad (15)$$

One must note that here we are referring to the relationship between efficiency and the frictional coefficients to be linear. The relationship between efficiency and magnetic field strength is squared in this same region. As with angular velocity, as η approaches 100%, these linear relationships break down. The efficiency converges to 100% for all values of frictional coefficients provided that the relative strength of the magnetic field is high enough. Also as in section III-A, the bounds of these linear relationships are directly proportional to their respective frictional coefficients. At higher levels of friction, a greater magnetic field strength is required to convert the rotational energy to electrical energy more quickly, thus reducing the impact of friction on the system.

C. Varying Disk Radius

For these simulations, we kept the moment of inertia (and, thus, the initial angular momentum) the same while changing the outer radius of the disk. If the moment of inertia is constant, we can equate the initial $m_o r_o^2$ to $m_f r_f^2$. As the mass of the disk is equal to the density ρ multiplied by the volume $\rho \pi r^2 h$ where r is the radius and h is the height or thickness of the disk, we can describe the following relationship:

$$\frac{r_f^3}{r_o} = \frac{h_o}{h_f} \quad (16)$$

As the resistance of the disk varies both with the radius of the disk and the thickness according to equation , we can see that the expected behavior is hard to predict.

Interestingly enough, there appears to be no relationship between the efficiency and the radius of the disk for a given moment of inertia. By observing figures 9 and 10, we can see that the efficiency is constant for a given frictional coefficient for all radius values simulated.

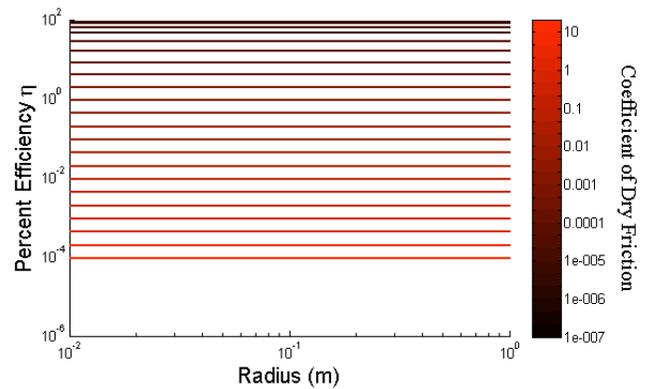


Fig. 9: Percent efficiency versus radius of the disk for different coefficients of dry friction (zero viscous friction). Note that there is no relationship between radius and efficiency, but that there is a linear relationship between efficiency and the inverse of the coefficient of dry friction until efficiency approaches 100%.

We feel that this is due to the fact that at the same rate that an increased radius will increase the *emf* of the disk, thus

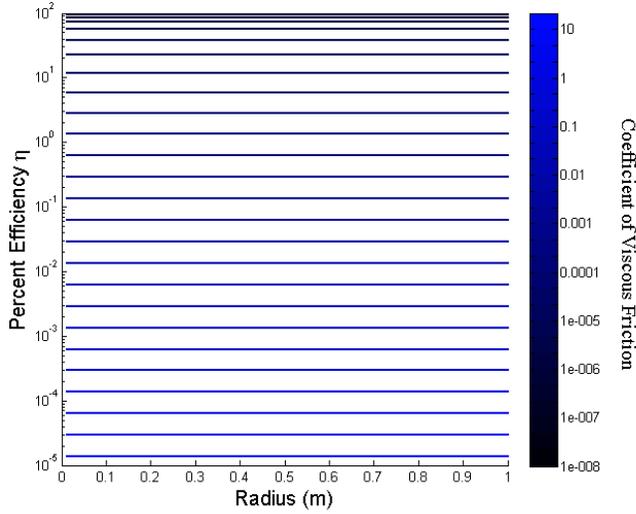


Fig. 10: Percent efficiency versus radius of the disk for different coefficients of viscous friction (zero dry friction). Note that there is no relationship between radius and efficiency, but that there is a linear relationship between efficiency and the inverse of the coefficient of viscous friction until efficiency approaches 100%.

decreasing the amount of time that the disk would spin, the increased radius increases resistivity (refer to section III-D), decreasing the amount of current generated. Unfortunately, attempts to derive the mathematical manipulations to support this surprising result have not yet been successful.

D. Varying Disk Resistivity

If we examine the relationship between resistivity and efficiency of the disk for a given value of the coefficient of dry or viscous friction, as seen in figures 11 and 12, we can see that the general relationship between efficiency and resistivity has, as in the case of many other characteristics of this system, a linear (in the log scale) and an asymptotic region. However, in this case, the linear region begins at a certain lower bound, rather than at zero, and continues to infinity. There is, as usual, a linear relationship between efficiency for a given resistivity and the frictional coefficients:

$$\eta(\rho_e) \propto \frac{1}{\alpha} \quad (17)$$

$$\eta(\rho_e) \propto \frac{1}{\beta} \quad (18)$$

The lower bound of the linear region of the logarithmic line is determined by the inverse of the frictional coefficient. As such, we can express the bounds of equations 17 and 18 as:

$$[\rho_{e_{min}}, \infty) \quad \dots \text{where} \quad \rho_{e_{min}} \propto \frac{1}{\alpha} \quad (19)$$

$$[\rho_{e_{min}}, \infty) \quad \dots \text{where} \quad \rho_{e_{min}} \propto \frac{1}{\beta} \quad (20)$$

If the resistivity lies below the lower bounds expressed by equations 19 and 20, the efficiency will be nearly 100%. As the frictional components decrease, this lower bound increases. For coefficients of dry and viscous friction below 1 (when only one form of friction is present) when considering a copper disk, the efficiency will approach 100%. For combinations of dry and viscous friction, the necessary coefficient of friction values for near-perfect efficiency will decrease. It should also be noted, however, that if we increase the angular velocity of the disk (which was for figures 11 and 12 only 10 radians per second), the effect of dry friction can be eliminated (refer to section III-A). This will not change the effect of viscous friction, however.

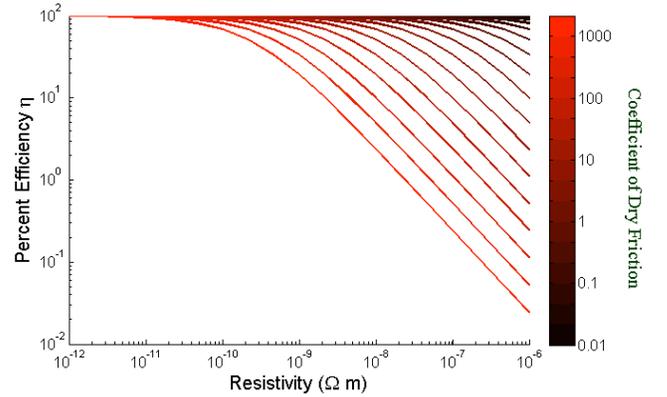


Fig. 11: Percent efficiency versus resistivity of the disk for different coefficients of viscous friction (zero dry friction).

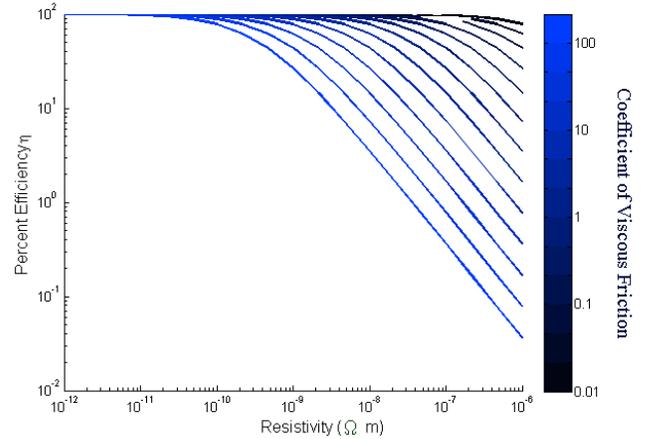


Fig. 12: Percent efficiency versus resistivity of the disk for different coefficients of dry friction (zero viscous friction).

IV. CONCLUSION

After much research of published writings devoted to the homopolar generator, we failed to find much quantitative information. Though some research papers included experimental results, we could not find any paper which discussed the implementation or even the conception of a comprehensive

model of the physical factors effecting the behavior of the homopolar generator. As such, our model provides a much more complete picture of the homopolar generator system than can be obtained from physical experimentation alone.

Our current results indicate the maximum efficiency can be obtained in several ways. If it were possible to create a disk with zero viscous friction, near-100% efficiency could be obtained at a high enough angular velocity, regardless of the coefficient of dry friction. However, if one were to account for the effects of eddy currents, high angular velocities and currents would reduce efficiency. It is also possible to create a high-efficiency generator by reducing the amount of time it takes for the rotational energy to be converted into mechanical energy. This can be accomplished by strengthening the magnetic field and by decreasing the resistance of the disk. By reducing the amount of time this conversion takes, the amount of time during which friction can act on the system is minimal.

However, in a generator, the efficiency is not the only important factor. Another consideration is the maximum amount of power that can be output. In the case of varying the magnetic field strength, a greater angular velocity would be required to increase power output. However, increased angular velocity would increase the effect of eddy currents on the system, decreasing efficiency. A possible solution to this problem is to use a superconductive disk. As shown in figure 3, if resistance approaches zero, the amount of time it takes the disk to stop would be zero. Under these conditions, the efficiency would be very high as the amount of time for friction to act on the system would be infinitesimal. Yet, because the currents would be extremely high, eddy currents would reduce the overall efficiency, preventing the system from reaching 100% efficiency.

In a future implementation of this model we plan to include the effects of eddy currents on the behavior of the homopolar generator. Upon inclusion of this factor, we will obtain the ability to test a homopolar generator under any conditions—assuming access to the necessary computing power—in order to further quantify the relationships between the various system characteristics. Through the quantification of these relationships, we hope to determine what levels of efficiency are possible with today's technologies, and what the theoretical maximum efficiency is for such a generator.

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